BOUNDS ON SATELLITE SYSTEM CAPACITY AND INTER-CONSTELLATION INTERFERENCE

Doron Rainish
SatixFy Ltd., 12 Hamada st. Rehovot, Israel 74140, Tel: +972-89393210,
Fax: +972-89393223, doron.rainish@satixfy.com

Abstract

The ever-increasing demand for data communication traffic, combined with the need for truly global coverage has spurred the introduction of new satellite systems with Low Earth Orbit (LEO) constellations, which are to operate independently, or in conjunction with Geosynchronous Earth Orbit (GEO) satellite fleets.

The paper takes a step further in analyzing the total capacity that can be achieved by a satellite system whereby terminals can receive, via multi-beams, several satellites. Thus, making it possible to re-use spectrum and uniformly share the load among multiple satellites.

As the number of satellites and constellations grows, the problem of coordination and spectrum sharing is becoming increasingly complex. In this paper, we analyze the impact of inter-constellation interference.

1. Introduction

The ever-increasing demand for data communication traffic, combined with the need for truly global coverage has spurred the introduction of new satellite systems with Low Earth Orbit (LEO) constellations, which are to operate independently, or in conjunction with Geosynchronous Earth Orbit (GEO) satellite fleets.

The paper takes a step further in analyzing the total capacity that can be achieved by a satellite system whereby terminals can receive, via multi-beams, several satellites. Thus, making it possible to re-use spectrum and share the load among multiple satellites. For this purpose, an analytic model for the capacity of LEO satellite system, based on multi-beam electronically steerable antennas was derived in this paper. The analytic model enables fast system parameters optimization such as number of antenna element, antenna radiation pattern, elevation angle, constellation height and more, and enables us to study the bounds on performance as a function of those parameters. It was shown that one big homogeneous constellation, that can be shared among several operators, can provide mankind capacity requirement for many years.

As the numbers of satellites and constellations grow, the problem of coordination and spectrum sharing is becoming increasingly complex. In this paper, we analyze the impact of inter-constellation interference. We show that this growing number may necessitate frequency sharing between the constellations.

The benefits of the multi-beam antenna, that grows with the number of its elements was demonstrated in increasing system capacity, decreasing the number of satellites, increasing capacity per DC power and reducing inter-constellation interference.

2. System Models

2.1 Antenna model

The antenna pattern has crucial influence on the inter and intra system interference. The usage of digital beam forming technology at the antennas creates new degrees of freedom in optimizing antenna pattern exploiting the accuracy of the phase shift, delay and gain that can be programmed at each antenna tap, that was not available with parabolic (metal) antennas and even with RF phase shifting antennas.

The antenna radiation pattern is designed by tapering the gains of the antenna patches. Generally, one can reduce the level of the side lobes ratio to the main beam (SLR) at the cost of increasing the beam-width [1]:

$$\Delta\emptyset_{3dB} = \frac{50.76°}{sin(\emptyset_0)}\frac{\lambda}{Nd}\, b \qquad for\ 0° < \emptyset_0\ < 180° \tag{1}$$

where ϕ_0 is the steering angle, N is the (one dimension) number of antenna elements, λ is the wavelength and d is the element spacing in wavelengths. The broadening factor b depends on the tapering. With no tapering, b=1 and the SLR 13 dB.

In this paper, we chose the Dolph-Chebyshev [1] which results in:

$$b = 1 + 0.636\left[\frac{2}{SLRa} cosh\left(\sqrt{acosh^2(SLRa) - \pi^2}\right)\right]^2 \tag{2}$$

where SLRa is the SLR in linear units. That is SLRa=$10^{SLR/20}$. Figure 1 shows an example of the antenna pattern for d=0.5, N=50 $\emptyset_0 = 80°$ and SLR=50 dB.

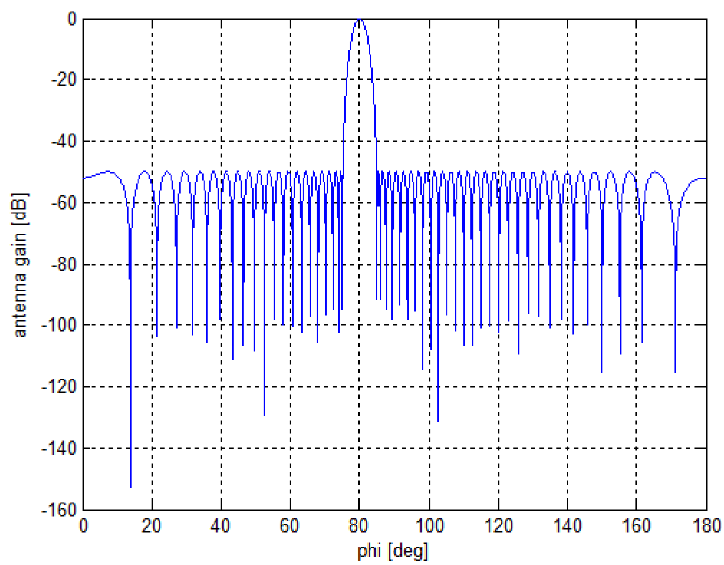

Figure 1 Dolph-Chebyshev antenna radiation pattern with N=50 d=0.5 SLR=50 dB steered to 80 degrees

2.2 Antenna Patch model

The antenna patch model assumed in this paper is given by:

$$G_{patch}(\emptyset) = 3 \cdot \cos(\emptyset)^p \tag{3}$$

Where ϕ is the steering angle (ϕ=0 at boresight) and p is the patch radiation factor (assumed here 1.5).

2.3 Interference Avoidance Model

The system in this paper is designed to guarantee inter-beam interference at the level of the transmit antenna SLR. For this end, we will define 2 additional beam-widths: service beam-width $\Delta\Phi_S$ and null to null beam-width $\Delta\Phi_{SLR}$. Since the communication protocol suggested here is based on per terminal beam hopping, a transmit antenna is always pointing to the destination ±pointing error. So, the service beam-width is defined here as the pointing error. The SLR beam-width is defined as the minimal beam-width outside of which, the SLR is higher than or equals to the antenna SLR. Figure 2 demonstrates these two beam-widths for the antenna in figure 1.

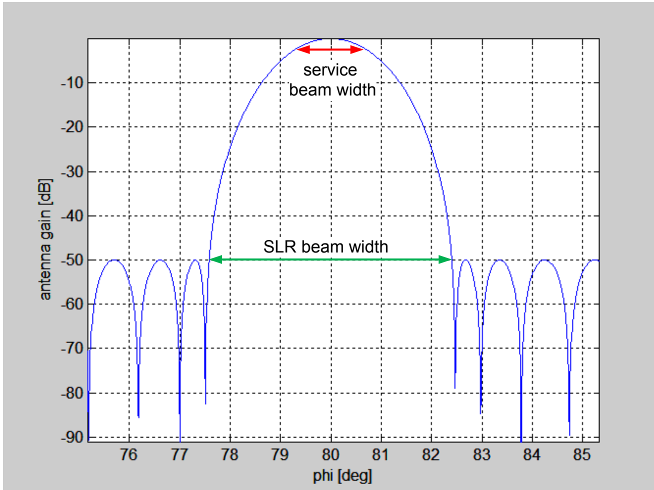

Figure 2 service and null to null beam-widths for the antenna in figure1

2.4 System Geometry

The system geometry is shown in figure 3 where S is the satellite, T is the terminal and O is the center of Earth. Re is the Earth radius, h is the satellite height above Earth, d is the distance between the satellite the terminal, α is the terminal elevation above the horizon, β is the satellite elevation from boresight (which is assumed to be directed towards Earth center).

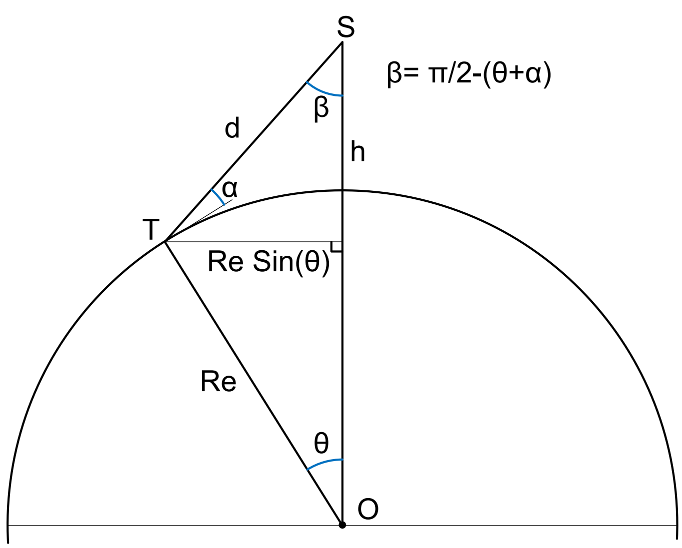

Figure 3 System geometry

There are some relations between the terms that will be used later [2]:

$$\theta = -\alpha + \cos^{-1}\left[\frac{Re}{Re+h}\cos\alpha\right] \tag{4}$$

$$\tan(\alpha) = \frac{(Re+h)\cos(\theta) - Re}{(Re+h)\sin(\theta)} \tag{5}$$

$$\beta = \frac{\pi}{2} - \alpha - \theta \tag{6}$$

$$d = Re\frac{\sin\theta}{\sin\beta} \tag{7}$$

From (4) and (6):

$$\alpha = \cos^{-1}\left\{\frac{Re+h}{Re}\sin(\beta)\right\} \tag{8}$$

2.5 Interference from terminal to satellites

The requirement here is that a terminal transmitting to a satellite will interfere with adjacent satellites at a level of at least the terminal SLR (SLR_term) below its EIRP. Therefore, the minimal angle between adjacent satellite the terminal sees must be equal or larger than the terminal transmit SLR beam-width $\Delta\Phi_{SLR\_term}$.

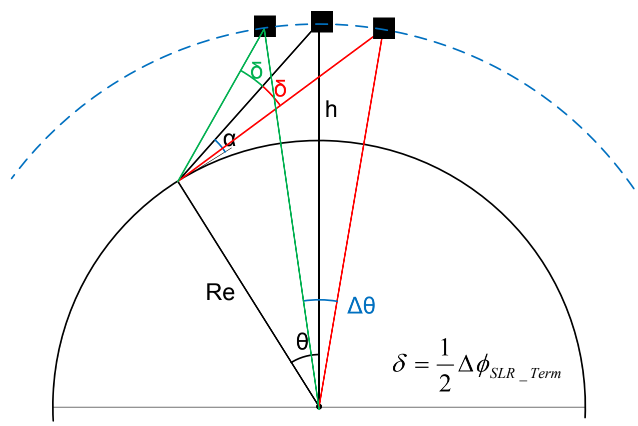

Figure 4 Adjacent satellite interference avoidance condition

The minimum satellite distance is computed as ½ Δθ which is the minimal center Earth angle between adjacent satellites. Δθ is larger when at the maximal elevation angle from boresight of the terminal α_0. Therefore:

$$\boldsymbol{\Delta\theta = \theta\left(\alpha = \alpha_0 + \tfrac{1}{2}\Delta\emptyset_{SLR\_term}\right) - \theta(\alpha = \alpha_0 - \tfrac{1}{2}\Delta\emptyset_{SLR\_term})} \tag{9}$$

Δθ can now be calculated using (4) and (9).

Once Δθ is evaluated, the number of satellites in a sphere cap defined by central Earth angle θ can be calculated. For this, we will use the satellite constellation described in [3] and shown in Figure 5.

The area of a sphere cap of radius R is given by:

$$A_c(\theta) = 2\pi R^2(1 - \cos(\theta)) \tag{10}$$

while the area of each hexagon is given by [3] and illustrated in Figure 5:

$$A_H = 6R^2\left[2\ \cos^{-1}\left(\frac{\sqrt{3}}{\cos\Psi}\right) - \frac{2\pi}{3}\right] \quad (11)$$

where Ψ is ½ Δθ. Therefore, the number of satellites Nc on the cap can be approximated as:

$$Nc(\theta, \Delta\theta) \cong \frac{\pi(1-\cos(\theta))}{3\left[2\cos^{-1}\left(\frac{\sqrt{3}}{\cos\left(\frac{\Delta\theta}{2}\right)}\right)-\frac{2\pi}{3}\right]} \quad (12)$$

The number of satellites on the cap fluctuates in time due to the satellites movement but it will be safe to assume that

$$Nc\left(\theta - \frac{\Delta\theta}{2}, \frac{\Delta\theta}{2}\right) < Nc\left(\theta, \frac{\Delta\theta}{2}\right) < Nc\left(\theta + \frac{\Delta\theta}{2}, \frac{\Delta\theta}{2}\right) \quad (13)$$

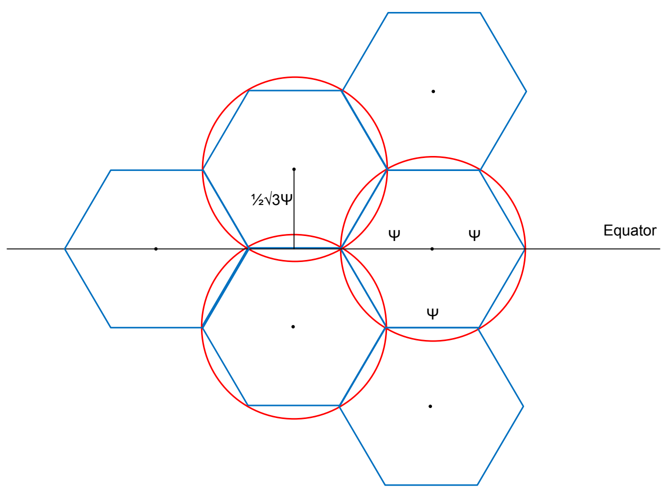

Figure 5 Constellation model from [3]

Note that at higher latitudes, the angle between orbits will decrease approximately as cos(Lat) and the terminal beam-width should be decreased approximately by the same factor. Therefore, the number of element in the terminal should be increased by approximately 1/cos(Lat). For example, at latitude=45º , a 41% increase in number of elements is required. Alternatively, the effective number of satellite at a specific latitude is decreased on the average as cos(Lat) (more and more satellites are de-activated when approaching the polar in order to keep minimal angle between satellites).

2.6 Interference from satellite to terminal

When a satellite is transmitting to terminals, it must keep at least $\Delta\emptyset_{guard\_sat} = \Delta\emptyset_{SLR\_sat} + pointing\ error$ between beams in order to guarantee that inter-beam interference will be lower than or equal to SLR_{sat}.

The number of beams a satellite can transmit simultaneously into its coverage area to randomly located terminals, obeying the above restriction is a random variable that can be calculated as follows. Let Ac be the satellite coverage area on Earth and FP be the footprint of the satellite beam of beam-width $\Delta\emptyset_{guard}$, with area A_{FP}. The probability that a terminal will not be able to get service, when one beam is active, that is, be inside the footprint of the other beam is A_{FP}/Ac. The probability that it will be able to get service is thus 1-A_{FP}/Ac, and the probability that it will be able to get service given K beams are transmitted is thus:

$$Ps(K) = \rho^K \quad where\ \rho = \left(\frac{Ac - A_{FP}}{Ac}\right) \tag{14}$$

One strategy of the satellite is to allocate beams to terminals (according to some QoS criteria for example) until a terminal cannot get service. In this case, the probability that M beams will be used is given by:

$$P(K = M) = (1-\rho)\prod_{k=0}^{M-1}\rho^k \tag{15}$$

Which reflects M successes and one failure. The average number of beams for this strategy is thus:

$$\widehat{K} = \sum_{M=1}^{\infty} M \cdot P(K = M) \tag{16}$$

However, the satellite can adopt another strategy in which a terminal that cannot be served is deferred to a later time or to another satellite with overlapping coverage area. In this case, when allowing Nr rejections:

$$P(K = M|\,Nr) = \left((1-\rho)\prod_{k=0}^{M-1}\rho^k\right)\sum_{i_1=1}^{M}\sum_{i_2=i_1}^{M}\dots\sum_{i_{Nr-1}=i_{Nr-2}}^{M}(1-\rho)^{i_1}\cdot(1-\rho)^{i_2}\cdots(1-\rho)^{i_{Nr-1}} \tag{17}$$

3. Parameter Calculations

3.1 Footprint Calculation

The footprint of a satellite beam changes shape from nearly circular close to the nadir point (red point in Figure 6) to an ellipsoid according to the satellite steering angle as shown in Figure 6. For calculating the worst case average number of beams the satellite can transmit into its coverage area, we will use the larger foot print, that is, the foot print at maximal satellite elevation from nadir.

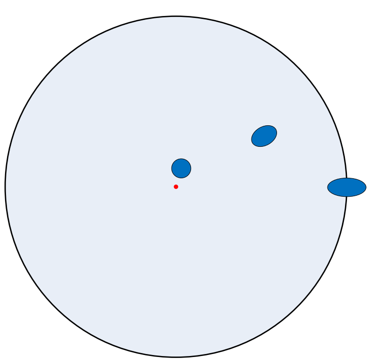

Figure 6 Beam footprint on Earth

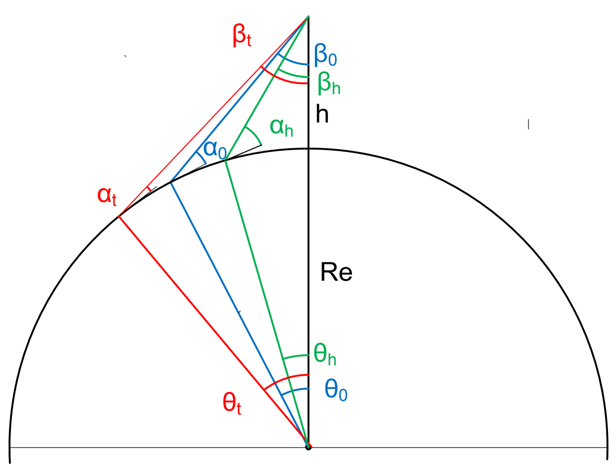

Figure 7 Calculation of the footprint size

*Figure 7 i*llustrates the footprint length calculation. β_0 is the maximal satellite elevation angle from nadir (α_0 is the maximal elevation angle from boresight) $\beta_t=\beta_0+½\Delta\phi_{guard}$ is the angle that reaches the footprint toe while $\beta_h=\beta_0-½\Delta\phi_{guard}$ is the angle that reaches the foot print heel. Using (8) α_t and α_h can be calculated and then θ_t and θ_h by (4) The footprint length is then calculated as:

$$foot\ print\ length = Re \cdot (\theta_t - \theta_h) \tag{18}$$

The width of the footprint can be calculated the same way using β0=0 and d is replaced by h. Finally, the area of the ellipsoid foot print area is calculated as:

$$A_{FP} = \frac{\pi}{4} foot\ print\ length\ \cdot\ foot\ print\ width \tag{19}$$

3.2 Averaged Interference level

The interference of terminals to a satellite is composed of the sum of transmissions arriving through different distances as shown in Figure 8. θ1=θ_s and $\theta_k=\theta_s$ where θ_s defines the cap the relevant interferers are on.

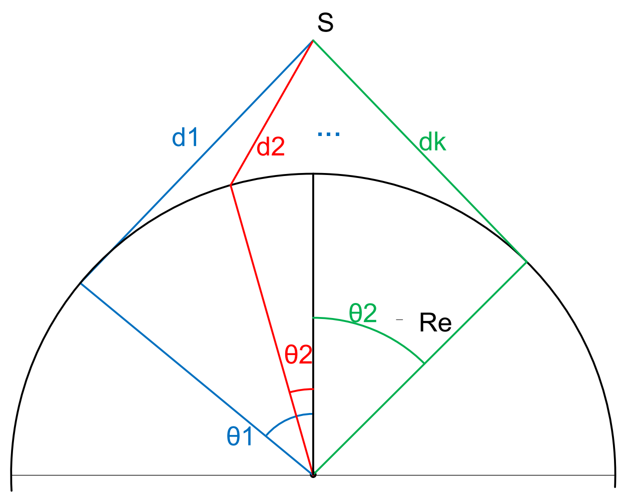

Figure 8 Average interference from terminals calculation

From Figure 3:

$$d_i^{\ 2} = [h + Re \cdot (1 - \cos(\theta_i))]^2 + Re^2 \cdot \sin(\theta_i)^2 \tag{20}$$

1/d² is averaged over the cap area:

$$A(\theta_s, Re, h) = \underset{\theta=0\ to\ \theta_s}{\text{mean}} \frac{1}{d^2} = \frac{Re^2}{2\pi Re^2 \cdot (1-\cos(\theta))} \int_{\vartheta=0}^{\theta_s} \int_{\varphi=0}^{2\pi} \frac{\sin(\vartheta)}{[h+Re\cdot(1-\cos(\vartheta))]^2 + Re^2 \cdot \sin(\vartheta)^2} d\vartheta d\varphi \tag{21}$$

$$A(\theta_s, Re, h) = \frac{1}{B \cdot (1 - \cos(\theta_s))} ln \frac{B \cdot (1 - \cos(\theta_s)) + h^2}{h^2} \tag{22}$$

where

$$B = 2Re \cdot (Re + h)$$

The additional loss due to this averaging can be calculated as

$$additional\ cap\ loss(\theta_s) = 10log_{10}\left(\frac{1/h^2}{A(\theta_s, Re, h)}\right) \text{ dB} \tag{23}$$

It easily deduced from Figure 3 that the maximal θ for which the terminal sees the satellite is given by:

$$\cos(\ \theta_{max}) = \frac{Re}{Re + h} \tag{24}$$

So, when considering all interferences from terminals in the visual cap, one should use θ_{max} in (23) resulting in:

$$additional\ cap\ loss(\theta_s = \theta_{max}) = 10log_{10}\left[\frac{2Re}{h\ln\left(\frac{2Re+h}{h}\right)}\right] \text{ dB} \tag{25}$$

We neglected here the influence of the antenna patch gain especially at low elevation. This creates a pessimistic view of the level of interferences. Numerical results show however that this approximation has a very small effect on the end result.

The same averaging calculation also holds for satellite to terminal interference when adding up all the interferences from all visual satellite to a terminal as shown in Figure 9.

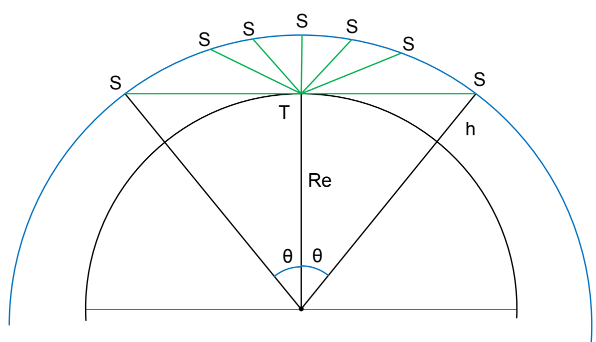

Figure 9 Average interference from satellites calculation

3.3 SNR Calculation

The downlink SNR is calculated as follows:

The average interference to a terminal from the satellite it communicates with is:

$$\left(\widehat{K} - 1\right) \cdot EIRP_{sat} \cdot SLR_{sat} \cdot additionalcap\ loss(\theta) \tag{26}$$

where $\widehat{K}$, the average number of transmitted beams, is taken from (16).

On top of it, all the satellites visible to a terminal will contribute:

$$EIRP_{sat} \cdot (N_{visual\ sats} - 1) \cdot \widehat{K} \cdot SLR_{sat} \cdot SLR_{term} \cdot additional\ cap\ loss(\theta_{max}) \tag{27}$$

where $N_{visual\ sats}$ is determined using (12) and (13) (where the upper bound is taken as a worst case) with $\theta=\theta_{max}$.

Similarly, the average interference to a satellite the from the $\hat{K}$ terminals it communicates with is:

$$\left(\hat{K}-1\right)\cdot EIRP_{term}\cdot SLR_{sat}\cdot additionalcap\ loss(\theta) \tag{28}$$

On top of it, all the terminals visible to the satellite will contribute:

$$EIRP_{term}\cdot(N_{visual\ term}-1)\cdot SLR_{sat}\cdot SLR_{term}\cdot additional\ cap\ loss(\theta_{max}) \tag{29}$$

where $N_{visual\ term}$ is determined using (12) and (13) (where the upper bound is taken as a worst case) with $\theta=\theta_{max}$. As worst case, the signal power is calculated according to:

$$\max path\ loss = 20\cdot log_{10}(frequency) + 20\cdot log_{10}(D_{max}) + 92.44 \text{ dB} \tag{30}$$

where D_{max} is the maximal distance between the satellite and the terminal calculated from (20) where $\theta=\theta_s$. The communication path loss is:

$$comm\ path\ loss = \max path\ loss + \max terminal\ patch\ loss + \max satellite\ patch\ loss \tag{31}$$

and the maximal patches' loss is calculated from (3) with maximal antenna and satellite elevations.

Eventually, the SNR is calculated according to:

$$SNR = P_{TX} + G_{TX} - back\ off - 10\cdot log_{10}(number\ of\ beams) - comm\ path\ loss + G\ over\ T_{RX} + 228.6 - 10\cdot log_{10}(symbol\ rate) \text{ dB} \tag{32}$$

The satellite capacity C_{sat} is then estimated as:

$$C_{sat} = symbol\ rate\cdot average\ number\ of\ beams\cdot log_2(1+SNR_{ef}) \tag{33}$$

Where the effective SNR, SNR_{ef}, is

$$SNR_{ef} = 10^{\frac{SNR-imp\_loss}{10}} \tag{34}$$

and imp_loss is the gap in dB between Shannon unconstrained capacity and the actual modem throughput.

3.4 Power Flux Density (PFD) Calculation

Both FCC [4] and ITU [5] restrict the maximal PFD at the Earth surface produced by a space station. Here we took a more conservative approach, adding up PFD from all satellites, including PFD resulting from side lobes, in a similar way as in the SNR calculations.

4. Numerical results

4.1 Constellation Capacity

In this section we will study some of the parameters of a system (capacity, figure of merit), as a function of antenna size. For all the calculations below, the following system parameters were assumed:

- Power amplifier back-off of 8 dB to accommodate multi-beam operation.
- Power amplifier efficiency of 40%
- 200 mW of DC power per antenna element. 20 dBm output power per element.
- If needed, the power amplifiers power was reduced so that the PFD on the ground is at least 6 dB below regulations ([4] and [5]).
- 50% of beam can be rejected (Nr= half of the number of beams, see (17))
- Terminal noise temperature is 24 dBK
- 2dB implementation loss between Shannon unconstraint capacity and actual capacity
- Capacity is calculated per single polarity 1 GHz BW (900M symbols per second)
- Terminal antenna tapering optimization range: SLR between 13 dB. (no tapering) and 30 dB
- Satellite antenna tapering optimization range: SLR between 13 dB. (no tapering) and 40 dB

Case study 1:

Maximal downlink capacity as a function of terminal antenna size and constellation orbit height above ground.

The terminal elevation angle optimization range is between 40º and 80º. We use equation 6 and 7 from [3] to calculate the number of orbits and number of satellites in an orbit (assuming polar orbits):

$$number\ of\ orbits = \left\lceil \frac{2\pi}{3\Psi} \right\rceil \tag{35}$$

$$number\ of\ satellites\ in\ orbit = \left\lceil \frac{2\pi}{\sqrt{3}\Psi} \right\rceil \tag{36}$$

with Ψ=1/2 $\Delta\theta$ where $\Delta\theta$ is calculated in (9).

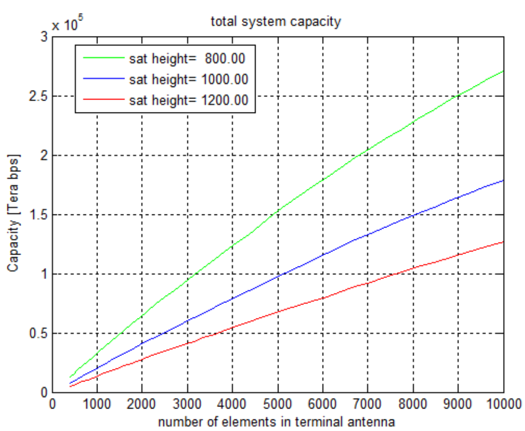

Figure 10 maximal downlink capacity as a function of terminal antenna size

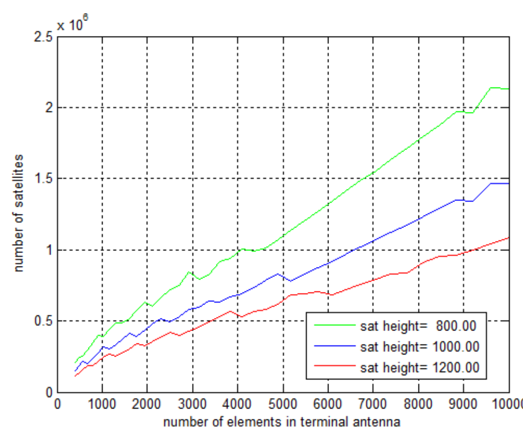

Figure 11 maximal number of satellites in the constellation as a function of terminal antenna size

Figure 10 and Figure 11 show the total system downlink capacity per 1 GHz and the number of satellites in the constellation respectively as a function of the terminal antenna size. The theoretical maximal number of satellites and the resulting capacity in a homogenous constellation is enormous. Many operators can share such a constellation.

Case study 2:

Downlink capacity with minimal terminal elevation angle (from the horizon) of 50º.

3 constellations are compared: 984 satellites at height 800 km, 680 satellites at height 1000 km and 493 satellites at height 1200 km. The number of elements in the terminal antenna is 400.

Figure 12 shows the total system capacity as a function of the number of elements in the satellite antenna for the three constellations while *Figure 13* shows the number of beams used. Figure 14 shows that the capacity per DC power figure of merit also increases as the number of satellite antenna elements increases.

When the number of satellite antenna elements is 4900, Figure 13 shows the total system capacity as a function of terminal antenna number of elements.

The number of antenna elements, both in the satellite and in the terminal has a substantial effect on the total system capacity.

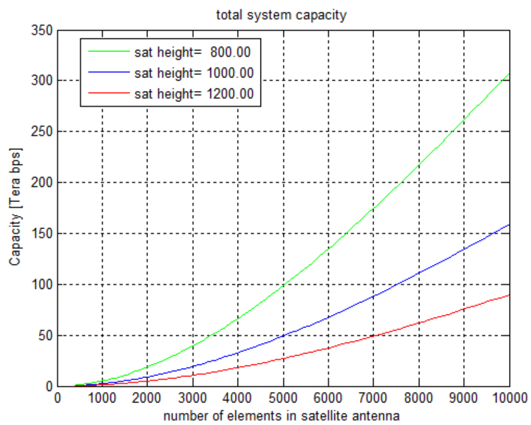

Figure 12 Total system capacity as a function of satellite antenna number of elements

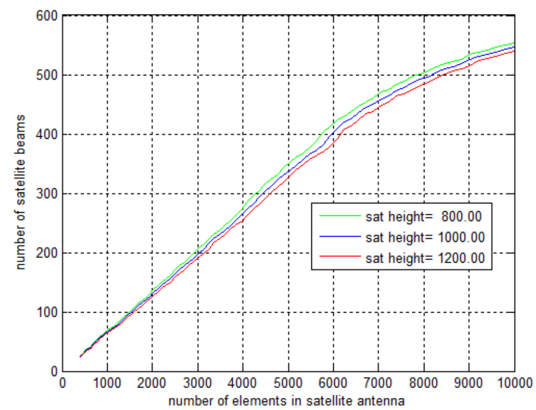

Figure 13 number of beams as a function of satellite

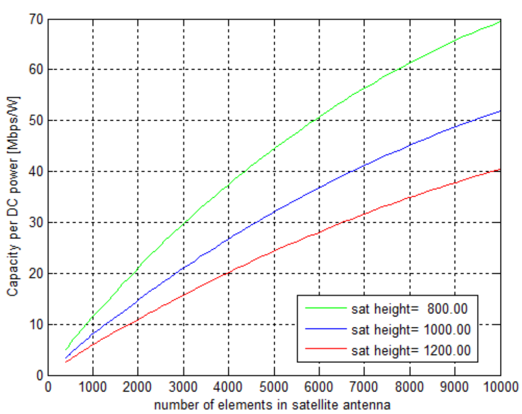

Figure 14 FOM as a function of satellite antenna number of elements

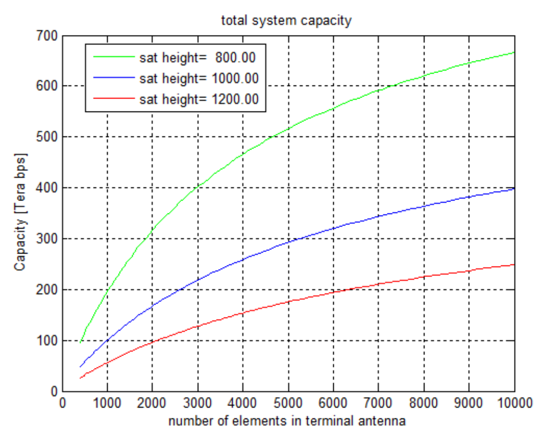

Figure 15 Total system capacity over 1GHz as a function of terminal antenna number of elements

Case study 3:

Same as case study 1 except that the satellite diversity is 4 (each terminal can communicate with up to 4 satellites).

The number of satellite are 14973 satellites at height 800 km, 10318 satellites at height 1000 km and 7705 satellites at height 1200 km.

Figure 16 shows the total system capacity as a function of the number of elements in the satellite antenna for the three constellations.

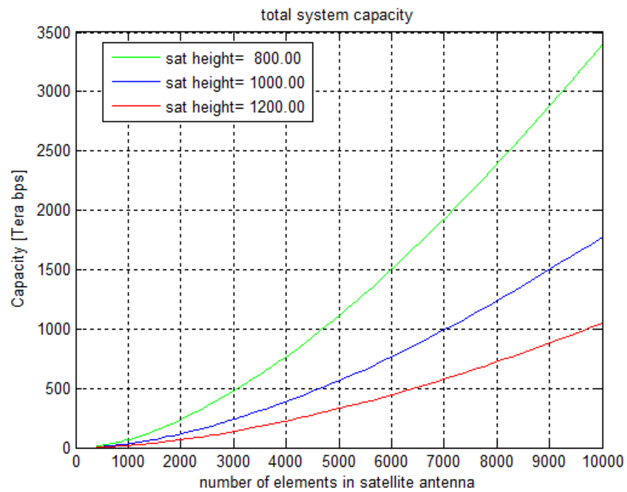

Figure 16 Total system capacity over 1GHZ as a function of satellite antenna number of elements

Case study 4:

The required capacity for the system is 100 Tbps. We would like to find the minimal number of satellites that comply with this requirement.

The allowed terminal elevation range is between 40º and 80º, and the satellite height is 1000 km. Figure 17 shows numerical results as a function of satellite and terminal antenna sizes.

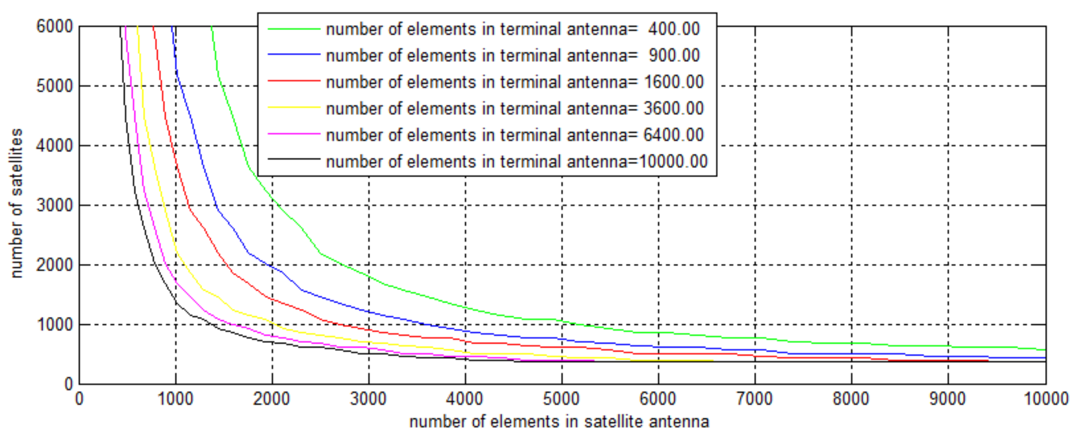

Figure 17 Number of satellites required to achieve 100Tbps system capacity as a function of satellite and terminal antennas number of element

Again, the number of antenna elements, both in the terminal and in the satellite antenna has a considerable effect on the number of needed satellites.

5. Inter-Constellation Interference Analysis

5.1 Down-Link Interference

The down-link inter-constellation interference scenario is illustrated in Figure 18. The interferer satellite (sat A) at height h_A is transmitting downwards a beam and hitting a footprint (Earth footprint) which is a function of h_A, the target terminal elevation α_A and the satellite beam-width. Any victim terminal inside this footprint cannot receive from a direction of the sky footprint, which is a function of the victim terminal transmit beam width $\Phi_{SLR\ termB}$. Note that some of the terminals in this footprint are not blocked since the blocker satellite is below their minimal elevation. The effective footprint size is thus a result of averaging over the terminal elevation angle α_A, taking into account the terminals minimal elevation. The effective sky footprint is a result of a similar averaging.

As a result of this transmitted beam, the victim terminal cannot receive from a fraction ρ_V of its total coverage area, where

$$\rho_V = \frac{Sky\ footprint\ area}{victim\ sky\ coverage\ area} \tag{37}$$

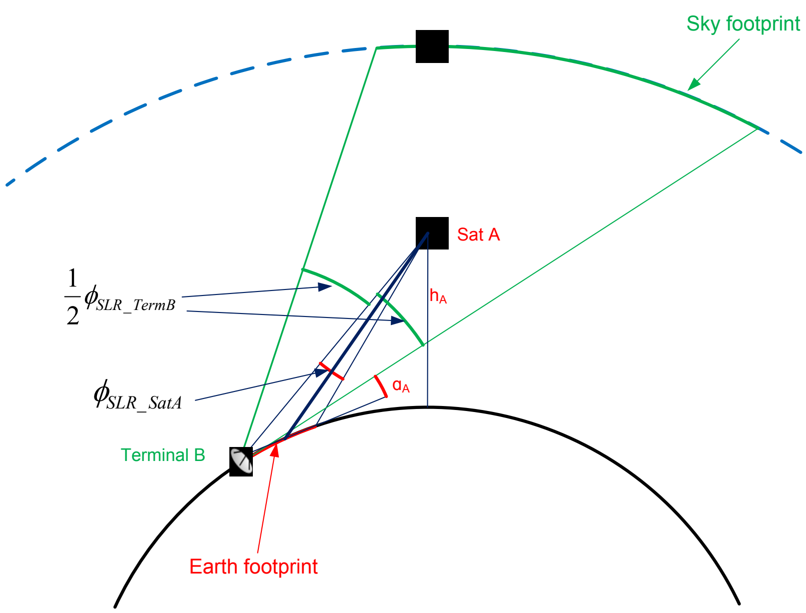

Figure 18 Down-link inter constellation interference scenario

Satellite A transmits N_J_SAT_BEAMS, each will create (presumably distinct) earth footprints so for non-overlapping satellites coverage areas, the fraction of Earth covered with these footprints is given by:

$$\rho_J = \frac{Earth\ footprint\ area \cdot N\_J\_SAT\_BEAMS}{Satellite\ A\ coverage\ area} \tag{38}$$

For overlapping satellite coverage area, that is, if a terminal for system A is communicating with Div_J satellites, the effective ρ_J is

$$\rho_J \sim \frac{Earth\ footprint\ area \cdot N\_J\_SAT\_BEAMS}{Satellite\ A\ coverage\ area}\ Div_J \tag{39}$$

since even if the footprints overlap, the directions of blocked receive sectors is different. The average blocked fraction of the victim system B can thus be approximated as:

$$P_{DNL\ block} \sim \rho_V \cdot \rho_J \tag{40}$$

Figure 19 shows numerical example of the down-link blockage. When there is only one interferer constellation with relatively low number of satellites (493) and low number of beams per satellite (200), adequate number of antenna elements, both in the victim terminal antenna and in the interferer satellite antenna can make the interference rather low. However, with 5 such constellations, and 500 beams per satellite, the interference impact is considerably higher as shown in Figure 20. Again, antennas with large number of elements minimizes the effect.

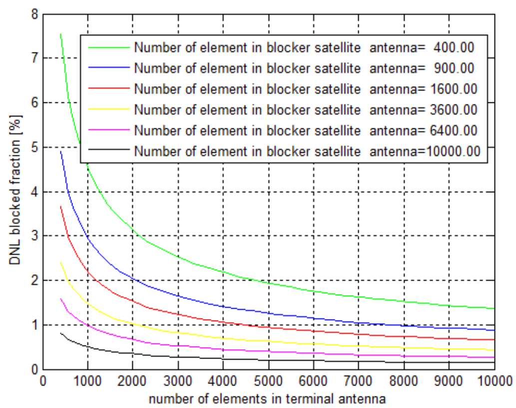

Figure 19 Down-link interference level as a function of terminal and satellite antenna number of elements for a interferer constellation of 493 satellites, 200 beams per satellite, at 1200 Km

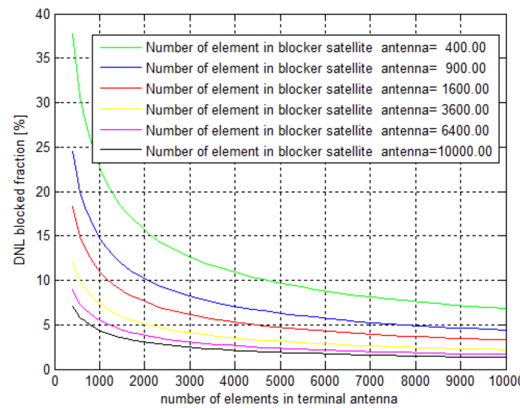

Figure 20 Down-link interference level as a function of terminal and satellite antenna number of elements for 5 interferer constellations constellation each of 493 satellites, 500 beams per satellite, at 1200 Km

5.2 Up-Link Interference

The Up-link inter-constellation interference scenario is illustrated in Figure 21. Terminal B is transmitting to satellite B and creates a sky footprint on the sphere of height $h_{A,}$, where system A satellites are. Note that some of the satellites in this footprint are not blocked since the blocker terminal is above their maximal elevation (from boresight). The effective footprint size is thus a result of averaging over the terminal elevation angle α_B, taking into account the satellites maximal elevation. The effective Earth footprint is a result of a similar averaging.

The average number of system A satellites in this footprint is given by

$$N\ sats\ A\ in\ sky\ coverage = \frac{total\ sats\ A \cdot terminal\ B\ sky\ coverage}{4\pi(Re+h_A)^2} = total\ sats\ A\,\frac{1-\cos(\theta_B)}{2} \tag{41}$$

where θ_B defines constellation B. The probability that a satellite from constellation A will be blocked can be approximated as

$$P_b = 1 - \left(1 - \frac{effective\ sky\ footprint\ area}{terminal\ B\ sky\ coverage}\right)^{N\ sats\ A\ in\ sky\ coverage} \tag{42}$$

If a satellite from constellation A is blocked, it cannot transmit into area of size of the effective Earth footprint around the blocker terminal B. Now, if in satellite A Earth coverage there are N_J transmitting terminals than on the average, $N_J \cdot P_b$ of them will block the satellite. The fraction of blocked area out of the total satellite A coverage area, which is the up-link system blocked fraction, is given by

$$P_{UPL\ block} = 1 - \left(1 - \frac{effective\ Earth\ footprint}{satellite\ A\ Earth\ coverage}\right)^{N_J \cdot P_b} \quad (43)$$

The average number of transmitting system B terminal in satellite A coverage area can be approximated as

$$N_J = \frac{number\ of\ system\ B\ satellites\ \cdot average\ beams\ per\ satellite \cdot satellite\ A\ coverage\ area}{Earth\ area} \quad (44)$$

Again, when there is only one interferer constellation with relatively low number of satellites (493) and low number of beams per satellite (200), adequate number of antenna elements, both in the interferer system terminal antenna and in the victim satellite antenna can make the interference rather low. However, when the interferer constellation is large (7705 satellites), the situation is much more severe as described in *Figure 23*.

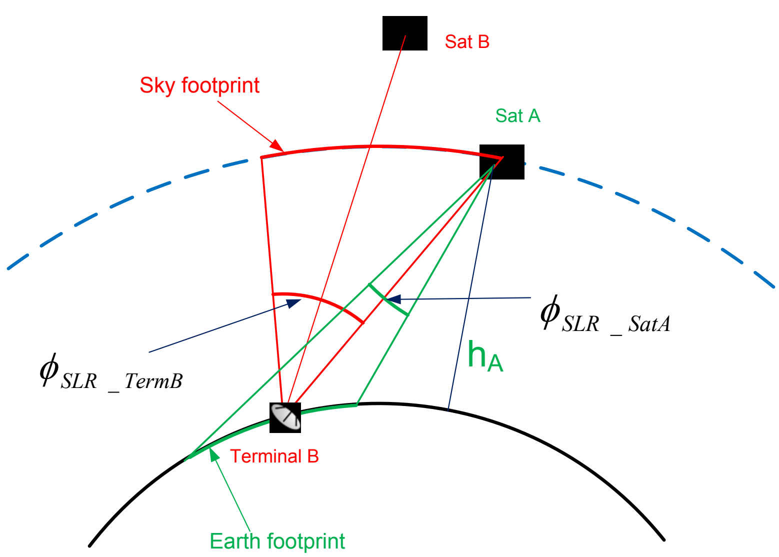

Figure 21 Up-link inter constellation interference scenario

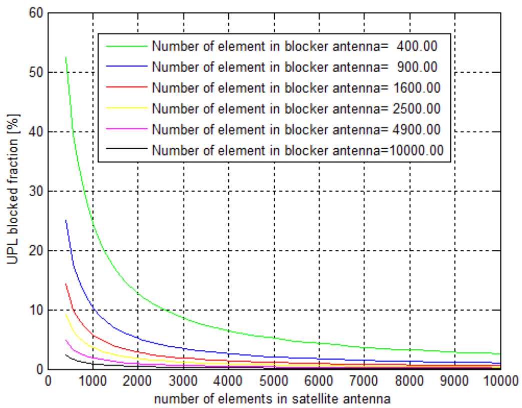

Figure 22 Uplink blocked fraction for 493 jammer satellites 200 beams each

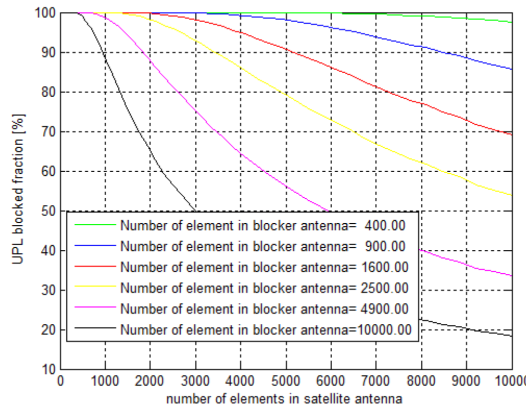

Figure 23 Uplink blocked fraction for 7705 jammer satellites 200 beams each

6. Conclusions

An analytic model for the capacity of LEO satellite system, based on multi-beam electronically steerable antennas was derived. The analytic model enables fast system parameters optimization such as number of antenna element, antenna radiation pattern, elevation angle, constellation height and more.

It was shown that one big homogeneous constellation, that can be shared among several operators, can provide mankind capacity requirement for many years. On the other hand, the inter-constellation interference grows quickly with the number of constellations and their size and may necessitate frequency sharing between the constellations.

The benefits of the multi-beam antenna, that grows with the number of its elements was demonstrated in increasing system capacity, decreasing the number of satellites, increasing capacity per DC power and reducing inter-constellation interference.

7. References

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3 Markus Werner, Axel Jahn, Erich Lutz and Axel Bottcher, Analysis of System Parameters for LEO/ICO-Satellite Communication Networks. IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, VOL. 13, NO. 2, FEBRUARY 1995 37 1

4 FCC CFR 47 25.208

5 Article 21 of the ITU Radio Regulations